STRUCTURAL IDENTIFICATION DURING AN EARTHQUAKE

by

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SYNOPSIS

Experiments and experience have shown that the behavior of large structures is nonlinear with the amplitude of the motion. The interaction effects of a flexible foundation also affect the frequency and damping characteristics of the structure, primarily in the first mode. For most loads acting on civil engineering structures it would be very difficult to determine the parameters associated with these phenomena. For earthquake loads, however, such information may be obtained from the relatively large response. A numerical method is developed to identify these from time histories of the free field ground acceleration and the response accelerations of the structure. Simulated ground accelerations and corresponding responses in one dimension are considered for motion of a one-story building on a rigid foundation and on a flexible foundation. The Fast Fourier Transform is used to simulate the acceleration time history. Least squares and parametric differentiation are used for the identification.

INTRODUCTION

For nuclear power plants and other relatively stiff structures soil-structure interaction effects can influence the dynamic response of the structure, particularly in the lowest and most important mode. This effect is not always conservative (8) and is difficult to evaluate without soil properties.

Experimental observations have verified the fact that for tall buildings the frequency and damping of the fundamental mode are nonlinear with amplitude (7). Although ambient wind excitations can be used to obtain foundation parameters, nonlinear effects cannot be determined from low-level excitations. In the case of earthquakes, however, the resulting motions may be large and the interaction parameters or nonlinear effects can be obtained. This is particularly applicable to damping estimates which cannot be predicted from low-amplitude tests(5).

The major objective of system identification in structural dynamics is the incorporation of the estimated parameters in a new design or in the modification of an existing design. The response must be controlled within acceptable levels, and these parameters of the model together with the mathematical model are the tools used in this control.

A method is herein developed and applied to this identification problem in which the data are time histories of the free field ground acceleration and building response accelerations during an earthquake. Parameters for a linear structure on a flexible foundation are obtained in an example where classical normal modes do not exist (8). In a second application, nonlinear effects are identified for a nonlinear structure on

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a rigid foundation.

The data utilized in this study are simulated on a digital computer. The Fast Fourier Transform (FFT) algorithm is used to generate an acceleration record (11, 12). The free field ground acceleration is considered as a stationary random process with constant spectral density up to a cutoff frequency, which is higher than the resonant frequencies of the ground-structure system.

The techniques of nonlinear least squares combined with a modified Newton-Raphson scheme and parametric differentiation of the equations of motion is used to estimate the parameters. Thus analysis is based on the actual system rather than an equivalent linear system (4) or an equivalent single degree-of-freedom model (10). Analytical representation of the solution in terms of the parameters is not required. Furthermore, the methods yields an approximate confidence ellipsoid on the parameter estimates.

Horizontal accelerations of one story building supported at the surface of an elastic half-space are considered. Generalizations to other models are discussed.

SIMULATION

For purposes of verifying the identification method, a simulated earthquake acceleration record was generated on a digital computer. In the actual identification problem, the researcher would have the free field acceleration record and the acceleration time histories of the structure. The ground acceleration should be nonstationary (6). Envelope functions have been used to modify finite sums of sinusoidal terms (9, 11, 12) in order to obtain nonstationary series.

Through optimization of results of a structure with a natural period of three seconds and a damping ratio of two tenths, it was found that a time series obtained from a truncated white noise representation matched the appearance of, and had similar statistical characteristics as the El Centro (1940) earthquake records (3). Stationary time histories with flat frequency content have been used in the response study of a stiff structure excited by Coulomb friction from a moving foundation (4) and in the analysis of a multi-story structures (13).

The simulation method used herein utilizes the FFT algorithm (12). The ground acceleration at the j-th time point is given by

$$\ddot{u}_g(t_j) = \sqrt{2}\ \mathrm{Real}\left(\sum_{k=1}^{N}\sqrt{S(\omega_k)\Delta\omega}\ e^{i\phi_k} e^{i2\pi\, j\frac{k}{N}}\right) j = 1,2 \ldots N \qquad [1]$$

in which N is the number of points; and

$$t_j = j\,\Delta\, t \qquad [2]$$

$$\omega_k = k\,\Delta\,\omega \qquad [3]$$

$$N = \frac{2\pi}{\Delta t \Delta\omega} \qquad [4]$$

and $S(\omega_k)$ is the one-sided power spectral density at ω_k. ϕ_k are independent random phase angles uniformly distributed between 0 and 2π. RANDU of the IBM Scientific Subroutine Package is used to generate ϕ_k.

As the goal here is to verify the identification scheme, the time series was not rendered nonstationary. The real application of indentification should be for actual observed time histories and not simulated records.

IDENTIFICATION BY NONLINEAR LEAST SQUARES IN THE TIME DOMAIN

Nonlinear Least squares techniques have been used previously to evaluate parameters of a mathematical model from time series responses (1). Using this type of data, corrections to estimates of the parameters are obtained iteratively until a convergence criteria is satisfied. The objective function may be generalized to a likelihood function in the case of correlated errors or to a posteriori likelihood function in the case of a priori information (1). In this study the errors are uncorrelated and there is no a priori information on the initial estimates. Thus for Gaussian errors with identical variance these are maximum likelihood estimates.

The data observed in a one story building is the ground acceleration and the total (not relative) acceleration of the roof. Thus for N time points the sum of the errors squared between these data and the results of a particular model dependent on the parameters is

$$E(\theta) = \sum_{\ell=1}^{N} \left(y_{t_\ell}(\theta) - y_{D_\ell} \right)^2 \qquad [5]$$

in which subscript t and D refer to theoretical and data roof accelerations, respectively. The analytical term may be approximated by a Taylor series about initial estimates to the parameters.

$$y_{t_\ell}(\theta) \simeq y_{t_\ell}(\theta_o) + \sum_{j=1}^{k} \frac{\partial y_{t_\ell}}{\partial \theta_j} \delta\theta_j \qquad [6]$$

This is substituted into eqn. [5], and optimisation with respect to the k-parameter corrections is performed. There results the equation

$$\left\{ \delta\theta \right\} = [A]^{-1} \left\{ R \right\} \qquad [7]$$

in which

$$A_{ij} = \sum_{\ell=1}^{N} \frac{\partial y_{t_\ell}}{\partial \theta_i} \frac{\partial y_{t_\ell}}{\partial \theta_j} \qquad [8]$$

$$R_i = - \sum_{\ell=1}^{N} \frac{\partial y_{t_\ell}}{\partial \theta_i} \left(y_{t_\ell} - y_{D_\ell} \right) \qquad [9]$$

The quantities are evaluated at the current estimates of the parameters.

The basic problems in this modified Newton-Raphson scheme are the determination of good initial estimates, upon which convergence or divergence depends, and the calculation of the required partial derivatives.

These are discussed in the section on the numerical examples. This method yields approximate confidence ellipsoids on the parameters.

The variance-covariance matrix of the estimates is (1).

$$\text{Cov.}\ [\theta_i, \theta_j] = \sigma^2 B_{ij} \qquad [10]$$

in which

$$[B] = [A]^{-1} \qquad [11]$$

σ^2 can be estimated by (1)

$$s^2 = \frac{1}{N} \sum_{\ell=1}^{N} (y_{t_\ell}(\hat{\theta}) - y_{D_\ell})^2 \qquad [12]$$

in which $\hat{\theta}$ is the estimate of the parameters.

LINEAR STRUCTURE ON A FLEXIBLE FOUNDATION

The effects of ground-structure interaction should theoretically be modeled by a system of integro-differential equations (8). The assumption of constant coefficients to represent this phenomenon, rather than frequency dependent coefficients yield simpler expressions for the equations of motion. Furthermore, the results are not greatly affected if the coefficients are picked judiciously, that is near the first resonant frequency of the coupled system (9).

The interaction equation for a single story building may be written in the convenient form (9).

$$[M] \left\{\ddot{x}\right\} + [C] \left\{\dot{x}\right\} + [K] \left\{x\right\} = \left\{f\right\} \qquad [13]$$

in which (fig. 1)

$$\left\{x\right\}^T = [u \quad u_b \quad \phi] \qquad [14]$$

$$\left\{f\right\}^T = [\ -m\ \ddot{u}_g \quad -(m + m_b)\ddot{u}_g \quad -\ mh\ \ddot{u}_g] \qquad [15]$$

$$[M] \begin{bmatrix} m & m & mh \\ m & m+m_b & mh \\ mh & mh & mh^2 + I_t \end{bmatrix} \qquad [16]$$

$$[K] \begin{bmatrix} m\omega^2 & 0 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & B_1 \end{bmatrix} \qquad [17]$$

$$[C] \begin{bmatrix} 2m\xi\omega & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & B2 \end{bmatrix} \qquad [18]$$

m is the top mass, m_b, the base mass, I_t the sum of the centroidal moments of inertia of the two masses and h is the building height. ω is the undamped circular resonant frequency of the structure on a rigid foundation and ξ is the associated damping ratio. A_1, B_1, A_2 and B_2 are the interaction coefficients which depend on the soil medium. $\ddot{u}_g$ is the free field ground acceleration, u_b the horizontal displacement of the base and ϕ is its rotation. u is the relative displacement of the top mass to the bottom mass. Particular mode shapes are given in fig. 2,

This formulation was based on results of a rigid circular plate on an elastic half-space (2) which were assumed to hold for rectangular foundations. The three matrices are symmetric with all the coupling terms located in the mass matrix. Coupling terms in the ground interaction have been neglected(8).

The total horizontal acceleration measured at the top mass is

$$y = \ddot{u}_g + \ddot{u}_b + \ddot{u} + h\ddot{\phi} \qquad [19]$$

thus for $\ddot{u}_g$ independent of the parameters the quantities needed in Eqns. [8 and 9] can be obtained by differentiating the equations of motion Eqn. [13] with respect to the parameter and interchanging the order of differentiation (1)

$$[M]\left\{\frac{\partial \ddot{x}}{\partial \theta_i}\right\} + [C]\left\{\frac{\partial \dot{x}}{\partial \theta_i}\right\} + [K]\left\{\frac{\partial x}{\partial \theta_i}\right\} = \left\{\frac{\partial f}{\partial \theta_i}\right\} - \left[\frac{\partial M}{\partial \theta_i}\right]\left\{\ddot{x}\right\} - \left[\frac{\partial C}{\partial \theta_i}\right]\left\{\dot{x}\right\} - \left[\frac{\partial K}{\partial \theta_i}\right]\left\{x\right\} \qquad [20]$$

Thus, the same system of equations hold, only the forcing function changes. The identification procedure is now complete. First initial estimates of the parameters are obtained through knowledge of the structure and the ground characteristics. With these initial estimates a corresponding motion may be generated from Eqns.[13-18] utilizing the observed ground acceleration.

The objective junction, Eqn. [5] is then evaluated and compared to the convergence criterion. If this is satisfied the variance-covariance matrix of the estimates is obtained from Eqns. [10], [11] and [12]. If not, a new correction vector is calculated from Eqns. [7], [8], [9], [19] and [20].

For the linear system with constant coefficients the response can be calculated analytically in terms of the parameters (1). The method is applicable to nonlinear systems also as shown in the following section .

NONLINEAR STRUCTURE ON A RIGID FOUNDATION

A number of nonlinear models have been proposed to model the behavior of a structure during earthquake loading. For stiff compact structure, Coulomb damping was considered (4). In the case of large stresses, some type of plastic yielding is appropriate (3).

It has experimentally been observed that frequency and damping are nonlinear with amplitude (7). When the base motion and rotation are neglected in Eqn. [13] and nonlinear effects are added, a mathematical model having these characteristics is

$$m\ddot{u} + c(1+\gamma u^2)\dot{u} + k(1+\mu u^2)u = -m\ddot{u}_g \qquad [21]$$

For a softening spring, μ is negative. Parametric differentiation yields k-scalar equations

$$m\left(\frac{\partial \ddot{u}}{\partial \theta_i}\right) + c\,(1+\gamma\, u^2)\,\left(\frac{\partial \dot{u}}{\partial \theta_i}\right) + k(1+\mu\, u^2)\frac{\partial u}{\partial \theta_i} = -\,\frac{\partial m}{\partial \theta_i}\,(\ddot{u}_g + \ddot{u})$$

$$-\,\frac{\partial}{\partial \theta_i}\,(c[1+\gamma u^2])\,\dot{u} - \frac{\partial}{\partial \theta_i}\,(k[1+\mu u^2])\,u \qquad [22]$$

This model has frequency and damping ratio which depend on the amplitude of the motion. It is not suggested that this is the model that represents behavior of actual structures. Rather, this serves as a nonlinear model for which the identification method is applied.

NUMERICAL EXAMPLES

The motions and partial derivatives satisfy ordinary differential equations. A Gill modification of the fourth order Runge-Kutta numerical integration scheme is used to solve these equations. This is subroutine RKGS of the IBM Scientific Subroutine Package. With initial conditions this method will work for cases in which the time step changes and will reduce it automatically if the round-off errors become too large.

Additional Parameters

Before proceeding to the examples, a number of additional parameters have been added. First it is possible that the accelerogram has a d.c. offset, or a constant level in the results. This parameter, κ, is added to the others, thus giving errors with a mean of zero.

Initial conditions of the motion and its derivatives may be taken from the data, but, these are subject to error, and are also considered as additional parameters. The initial acceleration, $\ddot{u}_1$ is assumed to be independent of the parameters except for these initial conditions and the constant offset. Thus

$$\frac{\partial u_i}{\partial u_o} = 1. = \frac{\partial u_{b_i}}{\partial u_{b_o}} = \frac{\partial \phi_i}{\partial \phi_o} \qquad i = 1 \qquad [23]$$

$$\frac{\partial \dot{u}_i}{\partial \dot{u}_o} = 1. = \frac{\partial \dot{u}_{b_i}}{\partial \dot{u}_{b_o}} = \frac{\partial \dot{\phi}_i}{\partial \dot{\phi}_o} \qquad i = 1 \qquad [24]$$

$$\frac{\partial y_{t_i}}{\partial \kappa} = 1. \qquad i, = 1, 2 \ldots N \qquad [25]$$

All other initial sensitivities are zero.

"Initial Estimates"

The identification scheme presented here is used to verify the engineer's estimates or to obtain better estimates of the parameters. The initial estimates are critical for convergence. The mass, base mass, total mass moment of inertia, and building height can be estimated from structural drawings.

The damping, nonlinear terms, and constant offset are initially assumed small , as are the initial conditions for motion beginning before the earthquake acceleration reaches the structure. The frequency can be obtained from structural considerations. Initial estimates for the interaction coefficients A_1 and B_1 may be obtained from static loading. A_2 and B_2 are initially set equal to zero.

"Rigid Foundation"

An acceleration spectral density of .75 $(ft/sec^2)^2$ with a cutoff frequency of 2 hz was used to simulate the ground acceleration. The standard deviation of the resultant motion is 1.17 ft/sec^2 compared to a theoretical value of 1.22 ft/sec^2 .

The parameters of the simulated model, initial estimates, final parameters, and associated variances are given in Table 1, for the case of "perfect" data and "corrupt" data. Gaussian errors with a mean equal to zero and a standard deviation of .1 ft/sec^2 was used to corrupt the data. The best fit standard error is .1006 ft/sec^2 . Sixty four points representing eight seconds of data were fit until the variance of the errors changed by less than .000001. The roof mass is nondimensionalized to a value of one and considered as known. As shown in the Table, the parameter are close to the correct values and have small variance for the case of "perfect" data. For measuring equipment with relatively large error characteristics, however, the nonlinear parameter are not well estimated. A five second time record of the data (*) and best fit (-) are shown in fig. 3 along with the simulated ground acceleration (---).

"Flexible Foundation"

The parameters in the soil-structure interaction identification example correspond to the same structure on a soil with a density of 125 lb/ft^3 and a shear wave velocity of 300 ft/sec. Poisson's ratio of the soil is zero (9). The structure has a fixed-base undamped natural frequency of 1 hz and a damping ratio equal to .01.

The attempt to determine the parameters from observation of the roof acceleration only failed. The relative, base and rocking motions contained high frequencies in the response. The total roof acceleration did not. For the parameters shown in Table 2, the eigenvalues and eigenvectors of the system were calculated. Although the modes were not exactly in phase, they were very close to being normal modes. These approximate normal modes are shown in fig. 2, together with the associated eigenvalues. Although there are non-zero mode shapes for the two higher eigenvalues, these have large damping constants and have components which are 180° out of phase. Thus when these are added, as in the case of total horizontal motion, they cancel each other. For the case in which all the acceleration components are observed, the parameters may be estimated and are given in Table 2.

"*Discussion of Numerical Results*"

The mode shapes of fig. 2 help to explain the success of an equivalent one-degree-of-freedom model of the structure (10). The frequency of the first eigenvalue is smaller and the damping is larger than the corresponding structure on a rigid foundation. The large eigenvalues have large decay characteristics associated with them. Furthermore, if only the total roof acceleration is observed,the motions due to these higher modes cancel each other.

Displacement, velocity, or relative displacement or relative velocity records can also be used for the identification of the parameters, but are not considered here. For tall structures, and in the case of vertical accelerations, each of the floor response accelerations should be observed for proper identification. Only a set of time histories is required, however, for estimation, unlike frequency domain methods.

The parameters are as good as the postulated model. Unfortunately, it is difficult to evaluate the adequacy of the model. Approximate variance of the estimate is given, however. For larger time series a sequential scheme may be more efficient than the one presented here. In particular, invariant imbedding should be considered (1).

CONCLUSION

A method is developed to identify structural and soil-structure parameters of a one story structure during an earthquake. It is based on observed free-field ground acceleration and acceleration of the roof. Approximate variances of the final estimates are obtained. The method deals directly with the system of differential equations and not with the solution. Thus it is applicable to nonlinear systems as well as to taller buildings in which the system is larger, or to situations in which there is vertical ground acceleration as well.

It was found that nonlinear parameters of a particular postulated model can be obtained from total roof acceleration for equipment with adequate error characteristics. For flexible ground, however, more data was required. When the base and rocking acceleration were also observed the parameters best describing the response can be estimated for this case.

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TABLE 1 RIGID FOUNDATION

f_u = 2 hz. Δt = .125 sec. T = 8 sec.

Parameter	Initial Estimate	"Perfect" Estimate	Variance	"Corrupt" (σ = .1) Estimate	Variance
c/m = .25*	.10	.25	$.53 \times 10^{-9}$	.256	$.76 \times 10^{-4}$
k/m = 39.48*	35.0	39.48	$.35 \times 10^{-7}$	39.49	$.29 \times 10^{-2}$
γ = 0.1	0.0	.10	$.20 \times 10^{-11}$	0.83	13.09
μ = 0.1	0.0	.10	$.57 \times 10^{-9}$	.007	$.48 \times 10^{-2}$
u_o = 0.0	0.0	0.0	$.90 \times 10^{-7}$	.001	$.11 \times 10^{-5}$
$\dot{u}_o$ = 0.0	0.0	0.0	$.17 \times 10^{-7}$	-.005	$.46 \times 10^{-4}$
K = 0.1	0.0	0.1	$.25 \times 10^{-4}$	.118	$.16 \times 10^{-3}$
σ		.014		.101	
# Iterations		13		10	

TABLE 2 FLEXIBLE FOUNDATION ("Perfect data")

f_u = 35 hz. Δt = .05 sec. T = 2.5 sec.

Parameter	Initial Estimate	Estimate	Variance
m = 3900*	3900		
h = 12*	12		
m_b = 5386*	5386		
I_t = $.4618 \times 10^{6}$*	$.4618 \times 10^6$		
ω = 6.283	5.8	6.277	$.17 \times 10^{-4}$
ξ = .01	.0001	.009	$.41 \times 10^{-6}$
A_1 = 21.25×10^6	19.50×10^6	20.62×10^6	$.53 \times 10^6$
A_2 = $.586 \times 10^6$	$.49 \times 10^6$	$.504 \times 10^6$	$.47 \times 10^2$
B_1 = $2257. \times 10^6$	$2200. \times 10^6$	$2368. \times 10^6$	$.32 \times 10^{10}$
B_2 = 10.38×10^6	11.00×10^6	2.12×10^6	$.83 \times 10^6$
σ		.082	
# Iterations		5	

* k/m, c/m, m, h, m_b, I_t were considered as known

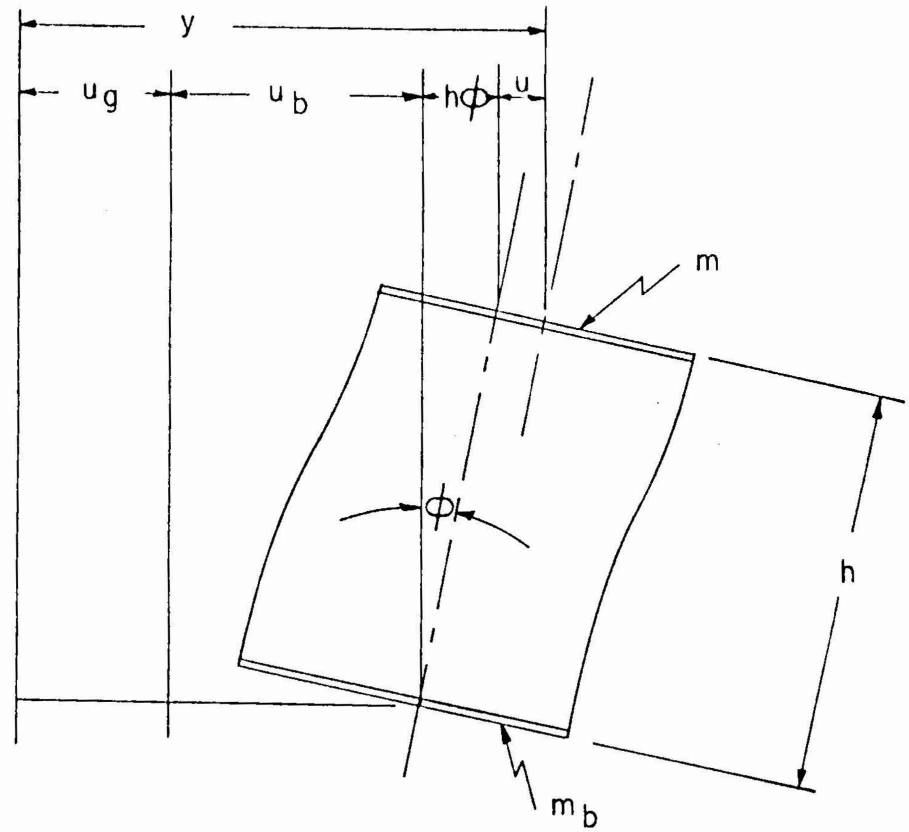

FIG. 1 SINGLE STORY BUILDING ON A FLEXIBLE FOUNDATION

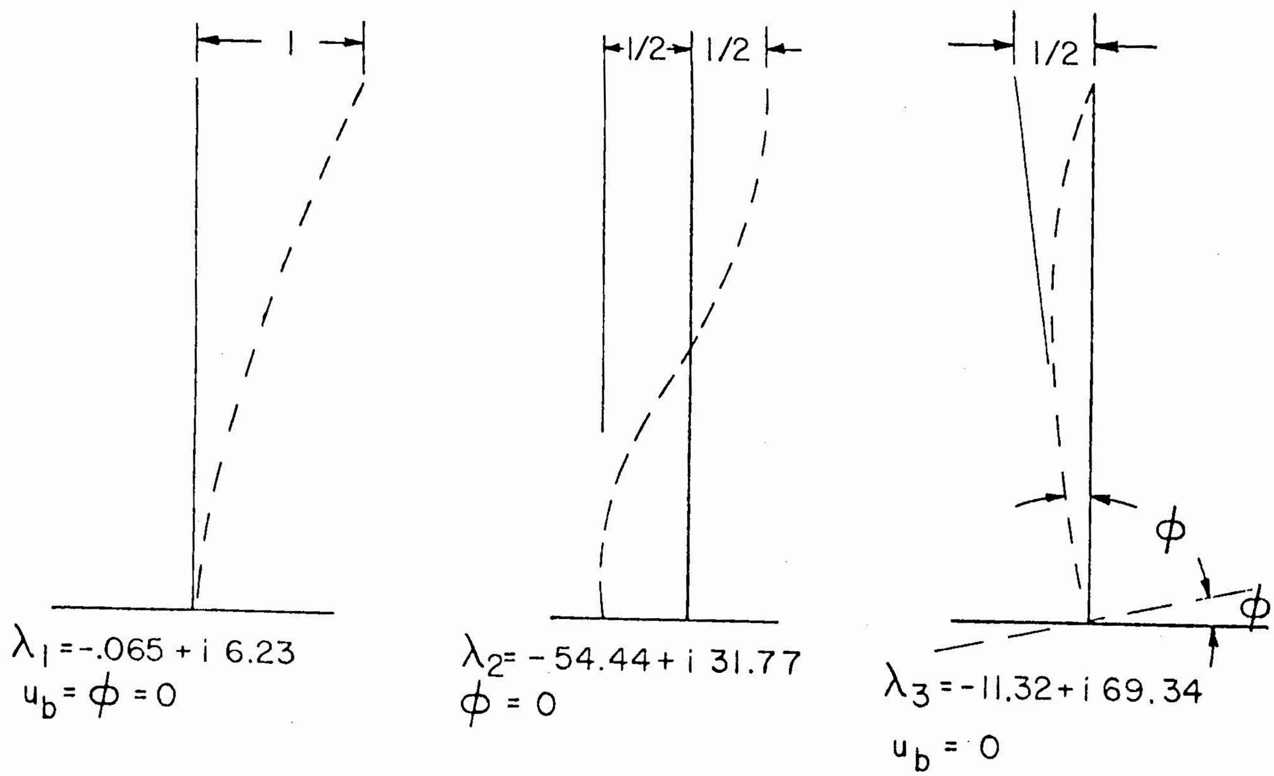

FIG. 2 APPROXIMATE NORMAL MODE SHAPES

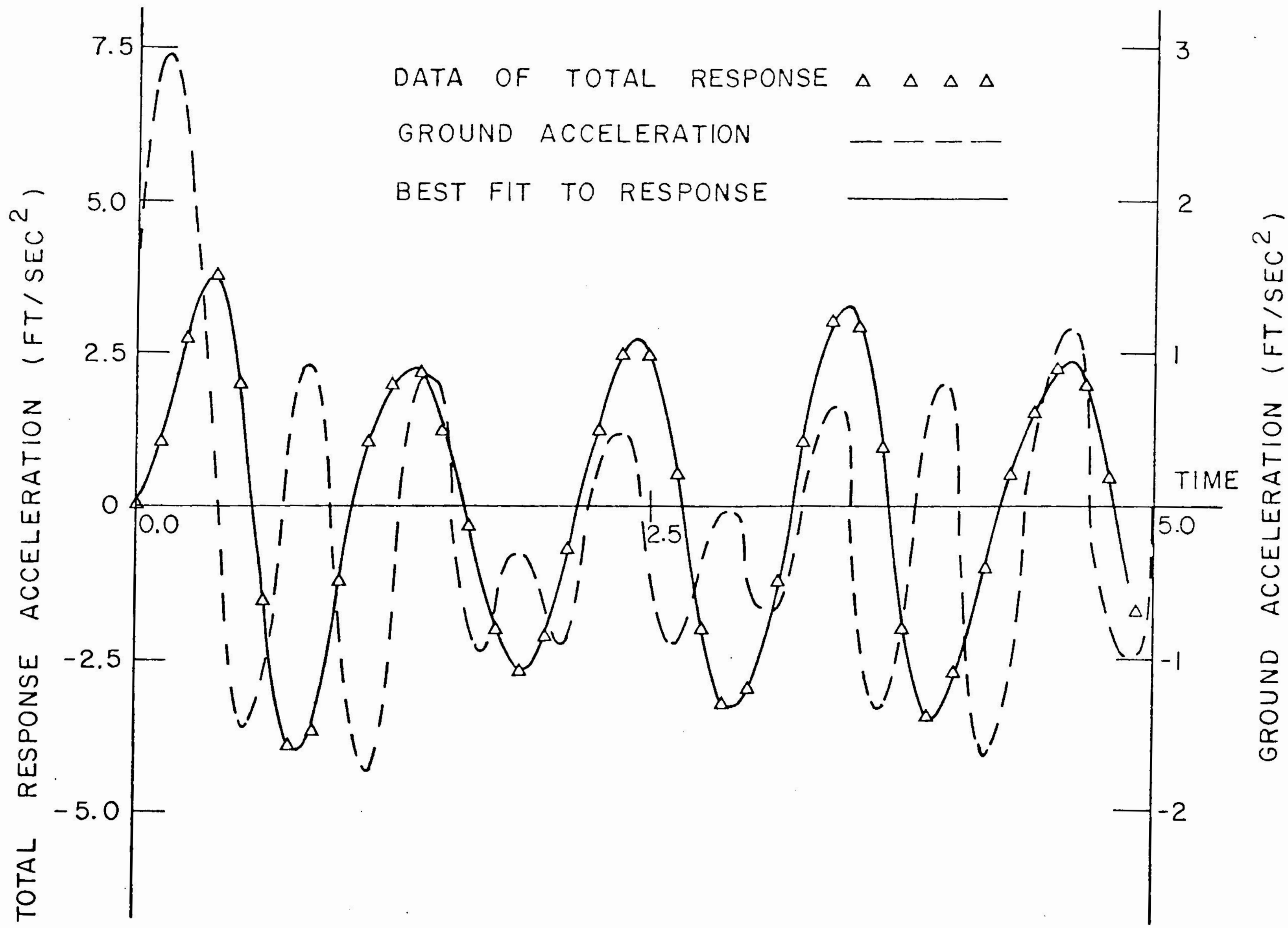

FIG. 3 ACCELERATION TIME HISTORIES